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FEATURES OF ELASTIC WAVE PROPAGATION IN A SYSTEM PLATE—LAYER—HALF-SPACE

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When solving questions for protecting supply-line engineering structures from the action of moving loads, information about the features of harmonic wave propagation in systems simulating real objects with their interaction with substructures and foundations taken into account can turn out to be useful. One of the models permitting clarification of the feature of dynamic interaction of structural elements is the system plate—layer—half-space. This latter (sometimes called a foundation) is considered rigid or deformable. Approximate methods for taking account of the pliability of the foundation in static problems are presented in [1].

Investigated below are the waveguide properties of the elastic systems plate—half-space, plate—layer, and plate—layer—half-space. Sliding contact is realized between the plate and the layer (half-space), while the layer and half-space are rigidly connected. Two foundation models are examined, exact (within the framework of elasticity theory) and approximation, the model of an elastic medium with one vertical displacement. Apparently, Rakhmatulin [2] first used this model in a problem not related to questions of structure interaction with a medium. Approximate equations are used in [3] to analyze the static and a number of dynamic problems. The dynamics of piecewise-homogeneous media was investigated in [4-6] by using this model and similar modifications. Information about the dispersion of harmonic waves in composite systems, obtained in [7-9], has a direct application to problems of geophysics, acoustodiagnostics, but leaves aside questions of strength and carrying capacity of the structures subjected to the action of waves being propagated in the surrounding medium. This is explained by the fact that the correspondence between the nature of free wave propagation and nonstationary processes under moving loads was not clarified. The fundamental merit in

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establishing such a connection is due to Slepyan, who analyzed the formation of quasistationary and resonance modes and related their parameters to dispersion parameters in separated spectrum domains [10]. Resonance phenomena in plates and shells surrounded by an acoustic medium are investigated in [10-13], and in a plane layer making contact with an elastic medium in [14, 15]. The results presented below make possible a qualitative analysis of resonance wave formation in structurally inhomogeneous systems without solution of the nonstationary problem.

Let us consider deformation of the media in the system. We introduce the notation: x is the longitudinal coordinate, y is normal, u and v are longitudinal and normal displacements, σ_x, σ_y, σ_{xy} are stresses. We denote the plate, layer, and half-space parameters by the subscripts 0, 1, 2, respectively.

The equation of dynamical plate bending on the basis of the classical model has the form

$$\ddot{v}_0 = (\rho_0 h_0)^{-1} R(x, t) - \varkappa^2 c_0^2 v_{0,xxxx}^{(\mathrm{IV})}, \tag{1}$$

where $\rho_0, h_0, c_0 = \sqrt{E_0/\rho_0(1-\nu_0^2)}$ are the density, thickness, and speed of sound in the plate, $\varkappa^2 = h_0^2/12$, and R is the reaction of the foundation

$$R(x, t) = \sigma_{yy}^{(1)}(x, 0, t). \tag{2}$$

We write the equations of layer and half-space motion in potentials

$$c_{1k}^2 \Delta\varphi_k = \ddot{\varphi}_k, \quad c_{2k}^2 \Delta\Psi_k = \ddot{\Psi}_k \quad (k = 1, 2) \tag{3}$$

(c_{1k} and c_{2k} are the expansion and shear wave velocities).

The displacements u_k and v_k and the stresses of interest to us are expressed in terms of the potential in a known manner:

$$u_k = \varphi'_{k,x} + \Psi'_{k,y}, \quad v_k = \varphi'_{k,y} - \Psi'_{k,x}; \tag{4}$$

$$\sigma_{yy}^{(k)} = \rho_k c_{1k}^2 \left[\varphi''_{k,yy} + (1 - c_{2k}^2/c_{1k}^2)\varphi''_{k,xx} - 2(c_{2k}^2/c_{1k}^2)\Psi''_{k,yx}\right]; \tag{5}$$

$$\sigma_{xy}^{(k)} = \rho_k c_{2k}^2 \left[\Psi''_{k,yy} - \Psi''_{k,xx} + 2\varphi''_{k,xy}\right]. \tag{6}$$

The boundary conditions for (3) are the sliding contact conditions for the plate and the layer

$$y = 0: \quad v_0 = v_1, \quad \sigma_{xy}^{(1)} = 0; \tag{7}$$

rigid contact of the layer and the half-space

$$y = h_1: \quad u_1 = u_2, \quad v_1 = v_2, \quad \sigma_{yy}^{(1)} = \sigma_{yy}^{(2)}, \quad \sigma_{xy}^{(1)} = \sigma_{xy}^{(2)}; \tag{8}$$

and damping at infinity

$$\varphi_2 = \Psi_2 = 0 \; (y \to \infty). \tag{9}$$

Equations (1)-(9) contain the formulation of the problem. We represent the desired functions in the form of sinusoidal waves traveling along the x axis [$\sim\exp(iq(x - ct))$, where c is the phase velocity, $q = 2\pi/\lambda$, λ is the wavelength]. After substitution into (3) and satisfaction of the boundary conditions (1), (2), (7)-(9) with the dependences (4)-(6) taken into account, we have a dispersion equation as a result of equating the determinant of the system of six linear algebraic equations to zero. Because of awkwardness, we represent writing this equation schematically by indicating the dependent on the problem parameters:

$$L(c, q; h_0, \rho_0, c_0, h_1, \rho_1, c_{11}, c_{21}, \rho_2, c_{12}, c_{22}) = 0. \tag{10}$$

This is a transcendental equation and the explicit dependence of interest to us $c = c(q, \ldots)$ is not obtained successfully from it. In the case of long ($q \to 0$) and short ($q \to \infty$) waves (10) can be simplified substantially and the asymptotic of the velocities can be found. A numerical analysis of the dispersion curves on an electronic computer causes no difficulties in principle. Represented below are results of such computations and the waveguide properties of the system plate–half-space, plate–layer–half-space, and plate–layer are analyzed as a function of the parameters. It is assumed that $\nu_0 = \nu_1 = \nu_2 = 0.25$ in all the computation modifications.

Plate–Half-Space

The dispersion equation corresponding to (10) under the condition that the layer and half-space parameters are identical ($c_{11} = c_{12} = c_1$, $c_{21} = c_{22} = c_2$ and $\rho_1 = \rho_2 = \rho$) has the following form (c_1, ρ, and h_0 are units of measurement):

$$\rho_0 q \sqrt{1-c^2}\,(c^2-\varkappa^2 c_0^2 q^2)\,c^2 + c_2^4 L_R = 0,\quad L_R = (2-c^2/c_2^2)^2 - 4\sqrt{1-c^2}\sqrt{1-c^2/c_2^2} \tag{11}$$

($L_R = 0$ is the Rayleigh equation).

For $q = 0$ (infinitely long waves) $c = c_R$ is the Rayleigh wave velocity for a free half-space, i.e., the presence of a plate of finite stiffness and mass does not influence the velocity of the long wave being propagated along the surface, which is a Rayleigh wave and "does not notice" the plate. In the shortwave spectrum ($q \to \infty$) we find from (11) the $c \sim c_0 q/\sqrt{12}$, which is a dependence governing the dispersion of bending waves in the plate in the absence of a foundation. This result is associated with the fact that the stiffness of the foundation surface layer with a thickness on the order of the wavelength tends to zero as q grows (while the stiffness of the plate is constant), and the influence of the foundation vanishes as $q \to \infty$. However, we note that the classical model of dynamical plate bending is not acceptable for a description of the shortwave ($\lambda \lesssim h_0$) spectrum and the space $q \to \infty$ is later not considered specially.

We obtain the next longwave approximation after $q = 0$ from (11) under the condition that q is small but finite $L_R + \rho_0 q (c/c_2)^4 \sqrt{1-c^2} + O(q^2) = 0$, from which we have the explicit expression

$$c = c_R(1-\alpha\rho_0 q + O(q^2)),\quad \alpha = \sqrt{1-c_R^2}\,(c_R/2c_2)^2\,\{(1+c_2^2-2c_R^2)\,[(1-c_R^2)(1-c_R^2/c_2^2)]^{-1/2} - 2 + c_R^2/c_2^2\}^{-1}. \tag{12}$$

As analysis showed, $\alpha > 0$ for $0 \leqslant \nu \leqslant 0.5$, consequently the phase velocity of the long waves decreases linearly from its limit value c_R as $\rho_0 q$ grows. The plate stiffness does not exert influence on the long wave dispersion (α is independent of the plate parameters), the asymptotic (12) agrees with that in a system where the plate is replaced by an inertial layer of zero stiffness ($c_0 = 0$).

The continuous lines in Fig. 1a-c ($\rho_0 = 0.5$, 1, 2, respectively) are the phase curves $c = c(q,\ldots)$ computed for different c_0 [in the case $c_0 = 0$ the asymptotic for $\rho_0 q \to \infty$ has the form $c \sim (\rho_0 q)^{-1/4}$], the dashes are the asymptotic (12), and the dash-dots are the shortwave asymptotic $c = c_0 q/\sqrt{12}$. As ρ_0 and c_0 diminish, the domain of the longwave spectrum broadens, where the dispersion is acceptably described by the approximation (12). As q increases (for constant ρ_0 and c_0) the role of the plate stiffness in the formation of the dispersion of the medium-wave spectrum is magnified, the phase curve smoothes out, reaches a minimum at the point q_*, c_* (the points are denoted by crosses on the curves) and later grows monotonically (the more rapidly, the greater the ρ_0 and c_0) until intersection with $c = c_2$ (the ordinates of the real phase velocities are bounded by this line since the dispersion equation has only complex roots for $c > c_2$). The presence of minimum points indicates the existence of critical velocities of load motion (equal to c_*) for this system, that excite resonance bending waves with frequency of the mode q_* in the medium-wave part of the spectrum.

The smaller the variability of the phase curve in the neighborhood of q_* (which corresponds to growth of the exponent n in the approximating dependence $c \approx (q - q_*)^n$, say, n even), the broader the wavelength spectrum forming the resonance perturbations, the more intensive their growth with time (asymptotically proportional to $t^{1-1/n}$ [10]) and the more rapidly is the asymptotic reached. Consequently, resonance modes in the medium-wave spectrum are more dangerous in the case of a relatively light and pliable plate. On the other hand, as c_0 grows the value c_* approaches c_R while q_* shifts into the longwave domain. This indicates the possibiltiy of superposition of surface ($c = c_R$, $q = 0$) and bending ($c = c_*$, $q = q_*$) growing waves and thereby substantial magnification of the resonance perturbations during motion of a normal load along the surface at a velocity that is incident in the interval (c_*, c_R). The question of which of these two mechanisms for the formation of perturbations that grow with time will predominate in each specific case can be solved during an analysis of nonstationary problems on the basis of numerical computations (e.g., by the scheme utilized in [15]).

As mentioned above, a simplified foundation model with one displacement [2, 3] was also considered simultaneously with the model (3), and to which one equation of motion for v corresponds:

$$\ddot{v} = c_1^2 v''_{,yy} + c_2^2 v''_{,xx},\quad \sigma_{yy} = \rho c_1^2 v'_{,y}. \tag{13}$$

Utilization of (13) in conjunction with (1) and condition (7) results in the dispersion equation

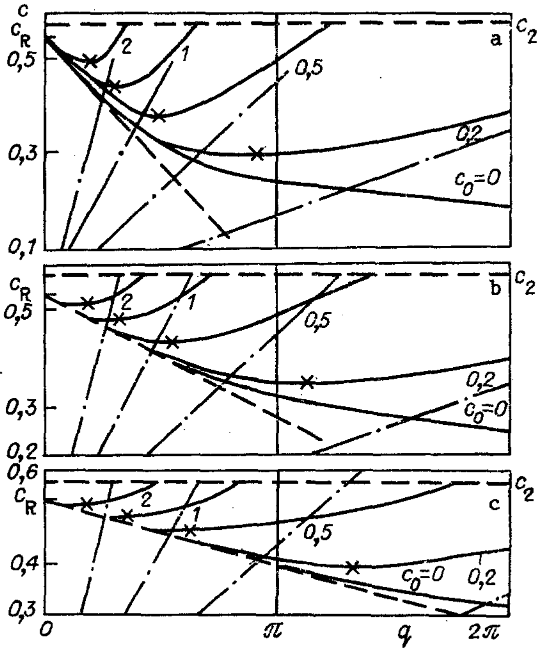

Fig. 1

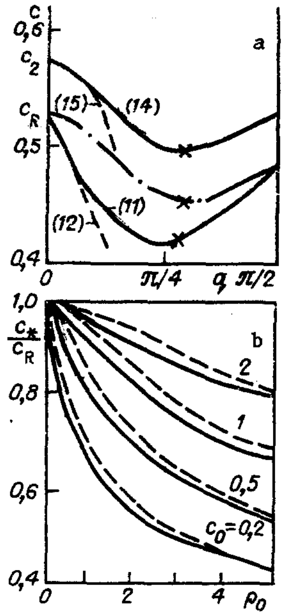

Fig. 2

$$c_2 \sqrt{1 - c^2/c_2^2} - \rho_0 q \left(c^2 - \varkappa^2 c_0^2 q^2\right) = 0. \tag{14}$$

Exactly as in the exact model, waves traveling at the velocity $c > c_2$ are radiated into the medium, however, in contrast to (12) the asymptotic values of the long wave velocities are different here

$$c = c_2 \quad (q = 0), \quad c \sim c_2\left(1 - \rho_0 c_2^2 q^2\right) \quad (|q| \leqslant \varepsilon). \tag{15}$$

As is seen from (15), the surface wave does not move at the Rayleigh but at the shear velocity, a decrease in the long wave velocity with the growth of ρ_0 has the same (linear) nature, however, it is weaker as q increases than for the exact model (not a linear but a parabolic dependence). These features govern the differences between the two models under investigation in the longwave spectrum. Presented in Fig. 2a is a comparison of the phase curves computed for $c_0 = 1$, $\rho_0 = 2.5$; the dashed lines are the asymptotics (12) and (15); the dash-dot line is the curve computed according to the approximate model (13) and corresponding to (14), where c_2 in these equations is replaced by c_R ($c_R \approx 0.92 c_2$ for $\nu = 0.25$). It is seen that such a replacement permits noticeable closure between the curves, and respectively, between the coordinates of the singular points. Represented in Fig. 2b is the dependence of c_* on ρ_0 and c_0, the continuous curve is the exact model, while the dashes are the approximate model with c_2 replaced by c_R. Let us note the following relative to the models being used: the difference in the curves in the neighborhood of the point q = 0 should result in a different degree of surface resonance wave growth [10, 15], whose amplitudes will be exaggerated in the approximate model, the behavior of the curves in the neighborhood of the minimum is qualitatively the same, the values of c_* do not differ substantially, and hence the description of the resonance bending waves in the plate should not have substantial distinctions.

Plate-Layer-Half-Space

In contrast to the preceding case, here an infinite number of roots of the dispersion equation exists, analogs of the modes of the free waves being propagated in a plane layer of finite thickness [10]. The first (lower) mode on a finite segment of the spectrum has real roots, the higher modes can be real or complex depending on the relationships between the system parameters. In the longwave spectrum ($q \to 0$) the asymptotic (10) takes the following form (c_{12}, ρ_2, and h_0 are units of measurement):

$$c_{22} L_R + q\left\{\rho_0 c^4 \sqrt{1 - c^2} + \rho_1 h_1 \left[\left(1 + \beta_{21}^2\right)\left(\beta_{12} + \beta_{22}\right) - 4\left(\beta_{21}^2 \beta_{12} + \beta_{11}^2 \beta_{22}\right)\right]\right\}, \quad \beta_{jk} = \sqrt{1 - c^2/c_{jk}^2} \quad (j, k = 1, 2). \tag{16}$$

The first component in the braces governs the influence of the plate (on the dispersion of long waves), the second, of the layer. The velocities of the asymptotics have the same structure as in (12), however, the coefficient α depends not on the layer parameters, and in contrast to (12) where $\alpha > 0$, can change sign.

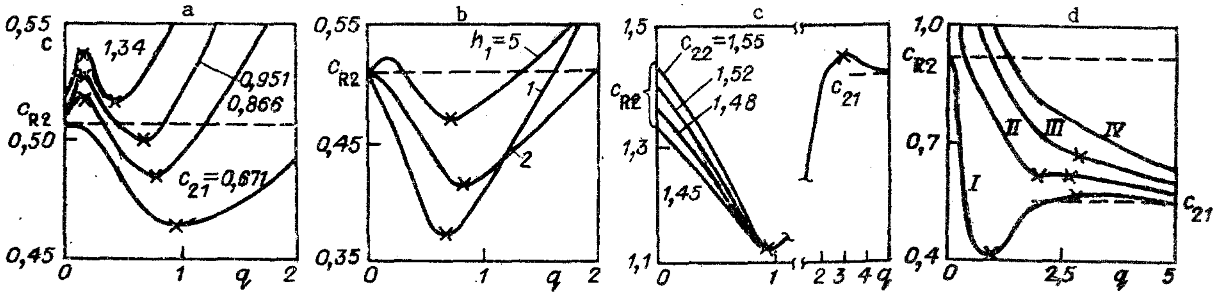

Fig. 3

For a relatively stiff and heavy layer with $c_{21} \geqslant c_{22}$ (or $c_{11} \geqslant c_{12}$, since $\nu_1 = \nu_2$) the value of α is negative and the velocities in the neighborhood of $q \leqslant \varepsilon$ exceed the asymptotic value at the point q = 0 that equals c_{R_2}. When the sign of α changes the component in the braces equals zero, the dependence c(q) in the neighborhood of q = 0 here becomes parabolic. This case is most favorable for the intensive development of surface resonance waves since the dispersion of the longwave spectrum is manifest substantially more weakly than for $\alpha \neq 0$ (analogous deductions are made in [7] for the layer–half-space system). For $\alpha < 0$ there are two singular points in the medium-wave part of the spectrum for the first mode (maximum and minimum), and one (minimum) for $\alpha \geqslant 0$.

Figure 3 shows phase curves computed for different values of the problem parameters: a) $c_{22} = 0.55$, $\rho_1 = 0.4$, $\rho_2 = 0.2$, $h_1 = 10$; b) $c_{21} = 0.95$, $\rho_1 = 0.4$, $c_{22} = 0.55$, $\rho_2 = 0.2$; c) $c_{21} = 1.41$, $\rho_1 = 0.75$, $\rho_2 = 1$, $h_1 = 5$; d) $c_{21} = 0.55$, $\rho_1 = 0.4$, $c_{22} = 1$, $\rho_2 = 0.5$, $h_1 = 5$.

For $c_{21} = 0.67$ and 0.95, $h_1 = 10$ and 2, the coefficient α from (12) equals zero and the asymptotic of the phase velocity c(q) in the neighborhood of q = 0 is parabolic while it is linear in the remaining cases. As is seen from Fig. 3b, as the bending stiffness of the layer approaches the plate stiffness ($h_1 = 1$), the dispersion is analogous to that in the plate–half-space system.

For a more light and pliable layer ($\alpha > 0$, $c_{21} < c_{22}$) there are two singular points in the medium-wave part of the spectrum, minimum and maximum, on the first mode curves. If the difference between c_{21} and c_{22} is comparatively small (Fig. 3c), then the phase curves in the domain $c < c_{R_2}$ differ slightly from those computed in the layer-half-space system analogously, have a singular point, a minimum; however the curvature of the firt mode changes sign in the shorter wave domain, and still another point is detected, a maximum whose ordinate exceeds c_{21} negligibly. Furthermore, as q grows, a change in curvature again occurs and the first mode tends asymptotically to c_{21} from above. Existence of the maximum is associated with the mutual influence of the models I-IV (Fig. 3d).

As computations showed, as the layer stiffness diminishes, singular points appear on the higher mode curves also. The second mode in the example represented in Fig. 3d has two points, a minimum and maximum, and a third, one (an inflection point with tangent parallel to the q axis). The presence of several singular points for different modes in a narrow spectrum (in this case the neighborhood of q = 2.5) indicates the possibility of superposition of different vibrations modes with nearby wave numbers in a dangerous (for the dynamic strength of the system) load velocity interval ($c_* = 0.55$-0.7). As q grows the higher nodes tend to c_{21}.

Plate–Layer and Comparison of Models

The roots of the dispersion equation are real in the whole spectrum in the plate–layer–rigid foundation system, and the number of modes is infinite. Analysis of the phase curves and the problem of determining the coordinates of the singular points are of practical interest since only a foundation layer of finite thickness is actually noticeably deformed. Consequently a natural question is the possibility of utilizing the approximate scheme for the dynamic computation of extended structures interacting with both structurally inhomogeneous and with a homogeneous foundation. Moreover, precisely this scheme is utilized for computation of the wave processes by numerical methods based on discretization of the spatial coordinates since the dimensions of the mesh domain are constrained by the volume of machine storage.

Curves 1-3 in Fig. 4a are computed for $\rho_1 = 1$, 0.75, 1.2 and $c_{12} = 0.58$, 0.84, 1.05, respectively. Exactly as in the preceding case for the light and pliable layer (Fig. 3c and d),

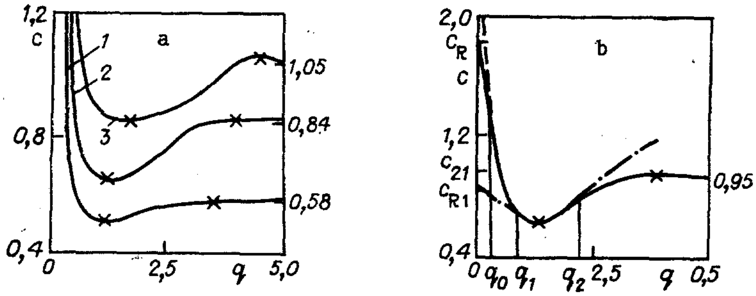

Fig. 4

two singular points exist in the medium-wave part of the spectrum, where the c_* of the maximum exceeds the velocity c_{21} (their ordinates are distinguishable in the scale of the graph), which is an asymptotic of the short ($q \to \infty$) waves. In the long wave domain the behavior of the curves is qualitatively different, here $c \sim \sqrt{f}/q$ ($f = c_{11}\sqrt{\rho_1/(\rho_0 h_0 h_1)}$ is the frequency of vertical vibrations of an inertial layer with the mass of a plate on a Winkler foundation with stiffness of the layer). The group velocities as $q \to 0$ tend to zero, i.e., in practice long waves are not propagated. This is the main distinction in the behavior of the wave pattern in the systems being compared.

Separation of the spectrum in which quantitative correspondence of the phase curves is observed is of interest. The solid line in Fig. 4b is the phase curve of the first mode for the plate–layer–half-space system with parameters governing a sufficiently stiff and heavy foundation ($h_1 = 5$, $\rho_1 = 1/3$, $c_{21} = 0.95$, $\rho_2 = 2$, $c_{22} = 4$), the dash–dot is for the system plate–half-space in which the half-space parameters agree with the layer parameters, and the dashes are for a plate–layer (here the half-space is considered stiff). The difference from the model with a deformable foundation is observed in this last case only in the domain of very long waves ($q < q_1 \approx 0.25$ which corresponds to $\lambda \gtrsim 25\ h_0$). In the medium-wave part of the spectrum ($q_1 \lesssim q \lesssim q_2$) including the singular point of the minimum, the plate–layer model can be utilized, which permits description of bending resonance waves in the case when the velocity of the moving load is less than the Rayleigh wave velocity for the layer.

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ANALYSIS OF SPALL FRACTURE IN SPECIMENS CONTAINING POROUS SPACERS

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UDC 539.375

Spall fracture can occur in metallic specimens under explosive and impact loading. It can be diminished or prevented entirely by using spacers of porous materials since they possess high energy absorption characteristics.

On the basis of numerical methods of the mechanics of a continuous medium, the influence of porous spacers on the spall fracture in cylindrical specimens subjected to explosive and impact loading is investigated in this paper.

1. The system of equations describing the behavior of a porous material in a two-dimensional axisymmetric formulation within the framework of the model of an elastic-plastic body has the form

$$\rho\dot{v} = \partial\sigma_z/\partial z + \partial s_{rz}/\partial z + s_{rz}/r, \ \rho\dot{u} = \partial s_{rz}/\partial z$$
$$+ \partial\sigma_r/\partial r + (2s_r + s_z)/r, \ \dot{V}/V = \partial v/\partial z + \partial u/\partial r + u/r,$$
$$\dot{E} = -pV + V[s_z\dot{e}_z + s_r\dot{e}_r + s_{rz}\dot{e}_{rz} - (s_r + s_z)\dot{e}_\varphi], \tag{1.1}$$
$$p = \frac{k_{0m}\left[1 - 0{,}5\Gamma_{0m}\left(1 - \frac{\alpha_0}{\alpha}V\right)\right]}{\alpha\left[1 - s_{0m}\left(1 - \frac{\alpha_0}{\alpha}V\right)\right]^2}\left(1 - \frac{\alpha_0}{\alpha}V\right) + \frac{\Gamma_{0m}E}{\alpha};$$

$$2\mu\dot{e}_r = \frac{D}{Dt}s_r + \lambda s_r, \ 2\mu\dot{e}_z = \frac{D}{Dt}s_z + \lambda s_z, \tag{1.2}$$
$$2\mu\dot{e}_\varphi = \frac{D}{Dt}s_\varphi + \lambda s_\varphi, \ \mu\dot{e}_{rz} = \frac{D}{Dt}s_{rz} + \lambda s_{rz}.$$

Here and below r, z are coordinates, u, v are velocity vector components along the r, z axes, σ_r, σ_z are stress tensor components, p is the pressure, s_r, s_z, s_{rz}, $s_\varphi = -(s_r + s_z)$ are stress deviator tensor components, E is the internal energy, $\dot{e}_r$, $\dot{e}_z$, $\dot{e}_{rz}$, $\dot{e}_\varphi$ are strain rate deviator tensor components, $V = \rho_{00}/\rho$ is the relative volume, $\rho_{00} = \rho_{0m}/\alpha_0$ is the initial density of the porous material, ρ_{0m} is the initial density of the host material under normal conditions, $\alpha = \rho_m/\rho$ is the porosity, α_0 is the initial porosity, ρ is the density, $\mu = \mu_{0m} \times (1-\xi)\left(1 - \frac{6K_{0m} + 12\mu_{0m}}{9K_{0m} + 8\mu_{0m}}\xi\right)$ is the shear modulus [1], μ_{0m}, K_{0m} are, respectively, the shear modulus and the multilateral volume compression, Γ_{0m} is the Grüneisen coefficient, s_{0m} is a material constant, Y_{dm}, Y_{0m} are the dynamic and static yield points, $\xi = (\alpha - 1)/\alpha$ is the relative pore volume, D/Dt is the symbol of the Jahmann derivative; all quantities without the subscript m refer to the porous material.

The parameter λ in (1.2) is determined by using the Mises flow condition for a porous material in the form

$$s_r^2 + s_z^2 + s_{rz}^2 + s_r s_z = \frac{1}{3}\left(\frac{Y_{Dm}}{\alpha}\right)^2. \tag{1.3}$$

The specific expression for λ is not presented since in the numerical method proposed for the solution of the problem [2], a procedure is used that reduces the stress to a flow circle, which is equivalent to the complete relationships (1.2).

The kinetic equation describing the compression of a porous material can be obtained from the solution of the equilibrium problem for a spherical pore subjected to an applied pressure [3]

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